[Answer key]

ACT 2020, MIDTERM #2
ECONOMIC AND FINANCIAL APPLICATIONS
MARCH 15, 2007

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You have 70 minutes to complete this exam. When the invigilator instructs you to stop writing you must do so immediately. If you do not abide by this instruction you will be penalised. All invigilators have full authority to disqualify your paper if, in their judgement, you are found to have violated the code of academic honesty.

Each question is worth 10 points. Provide sufficient reasoning to back up your answer but do not write more than necessary.

This exam consists of 8 questions. Answer each question on a separate page of the exam book. Write your name and student number on each exam book that you use to answer the questions. Good luck!

Question 1. Rover & Barkley Corporation (RBC Corp.) has decided to issue a mandatory convertible bond. The bond matures at time T, pays a 5% annual coupon on a \$1,000 notional face amount, and the share price at that time is denoted S_T. The mandatory convertible bond pays the bondholder 30 RBC shares at bond maturity if the share price is below \$40, it pays the bondholder $(30 \cdot 40)/S_T$ RBC shares at bond maturity if the share price is between \$40 and \$60 and it pays the bondholder 20 RBC shares at bond maturity if the share price is above \$60.

(1) [3 points] Draw a chart of the payoff at bond maturity to the owner of the RBC Corp. mandatory convertible bond. (As was done in class, your chart should not include the interest component of the bond.)

(2) [2 points] If it turns out that $S_T = 37$, compute the dollar amount of the payoff at bond maturity to the owner of the RBC Corp. mandatory convertible bond.

(3) [5 points] The President of RBC, Ms. Barkley, is also interested in the possibility of issuing a convertible bond. This bond would pay a 4% annual coupon on a \$1,000 notional face amount, and would permit the bondholder to receive the face amount of \$1,000 at maturity or 25 shares in RBC Corp., to be determined at the bondholder's discretion. Explain how this convertible bond may be viewed as a bond plus a call option and draw a chart of the payoff at bond maturity to the owner of this proposed RBC Corp. convertible bond. (As was done in class, your chart should not include the interest coupon component of the bond.)

Q2:

A strategy consists of longing a put on the market index with a strike of 830 and shorting a call option on the market index with a strike price of 830. The put premium is \$18.00 and the call premium is \$44.00. Interest rates are 0.5% per month. Determine the net profit or loss if the index price at expiration is \$830 (in 6 months).

(a) \$0
(b) \$23.67 loss
(c) \$26.79 gain
(d) \$28.50 gain
(e) The correct answer is not given by (a), (b), (c), or (d)

Q3:

What is the maximum loss that an investor can obtain over 6 months from a strategy employing a long 830 call and a short 850 call? Interest rates are 0.5% per month.

(a) \$6.80
(b) \$7.68
(c) \$9.24
(d) \$12.32
(e) The correct answer is not given by (a), (b), (c), or (d)

Q4:

Why would a manufacturer elect to use a long call strategy instead of a forward contract to hedge the risk associated with variable costs?

Q5:

A farmer sells 4 million bushels of corn at a spot price of \$2.10 per bushel. The total cost of production was \$9.2 million. The farmer has an effective tax rate of 25%. If the farmer entered into a futures/forward contract at a price of \$2.40 per bushel on 4 million bushels, what is the farmer's net loss or gain?

(a) \$100,000 loss
(b) \$800,000 loss
(c) \$300,000 gain
(d) \$400,000 gain
(e) \$0

Q6:

For the following problem assume the effective 9-month interest rate is 4%, and the FTSE100 forward price is 4,446. We assume for simplicity that the FTSE100 index does not pay dividends. You should use these premiums for FTSE100 index options with 9 months to expiration:

Strike	Call	Put
3,925	713.07	
4,025	653.62	248.81
4,275	520.47	356.04
4,325	496.46	
4,425	450.98	430.79
4,725	333.96	
4,775	317.11	633.46
5,025	243.19	799.92

Calculate the price of a long butterfly spread using the following call options: £3,925-strike call, £4,325-strike call, and £4,725-strike call.

INFORMATION FOR QUESTION 7 & 8:

S.O. extracts oil, with fixed costs of $20/barrel and variable costs of $8/barrel.

Plastics Corp. produces PET resin. It buys oil and manufactures the resin. One barrel of oil can be used to produce two units of resin. One unit of resin sells for 0.4 times the price of one barrel of oil, plus $15. The fixed cost per unit of resin is $7 and the non-oil variable cost is $3.30 per unit of resin.

The one-year forward price of oil is $42/barrel. The one-year continuously compounded interest rate is 4%. One year oil option prices are shown in the table below:

Strike	Call Price	Put Price
35	$8.085	$1.359
40	$5.106	$3.185
42	$4.174	$4.174
45	$3.032	$5.915
50	$1.711	$9.397

Q7:

If Plastics Corp. chooses to hedge their oil price exposure with futures so that they are completely immune to changes in the oil price, how many oil futures should they buy per unit of resin produced? What is its estimated profit one year from now? Construct graphs illustrating both unhedged and hedged positions.

Q8:

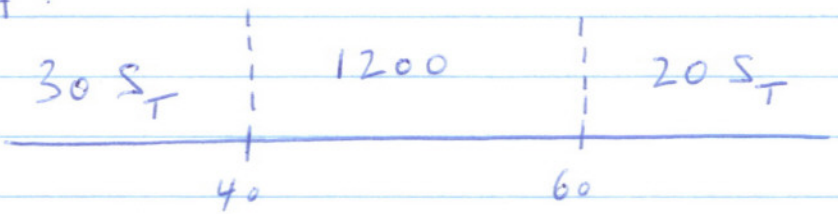
Compute the estimated profit in one year if Plastics Corp. buys one call option per ten units of resin with a strike of \$35(\$45). Draw a graph of profit in each case.

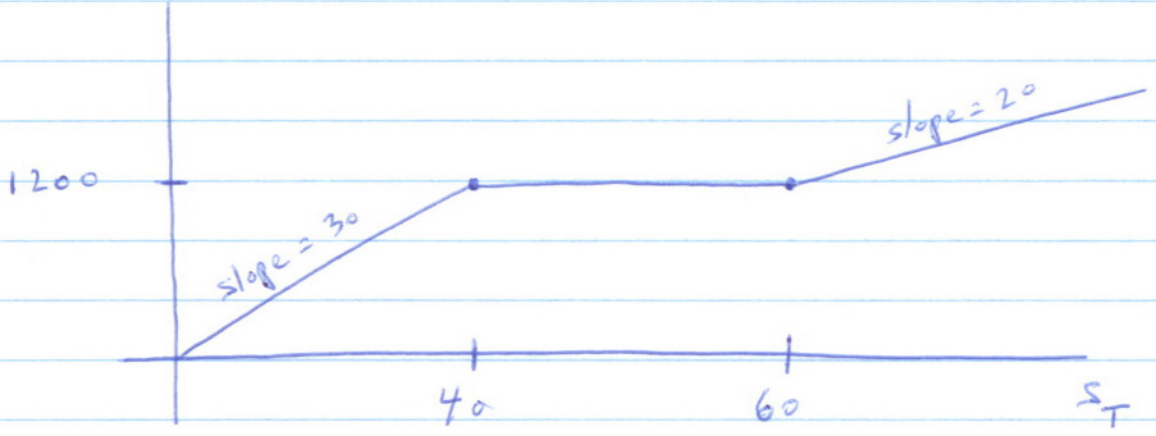
[5 pts for $K = 35$
5 pts for $K = 45$]

[ACT 2020 Midterm 2 Solutions]

Q1:

(1) Payoff:





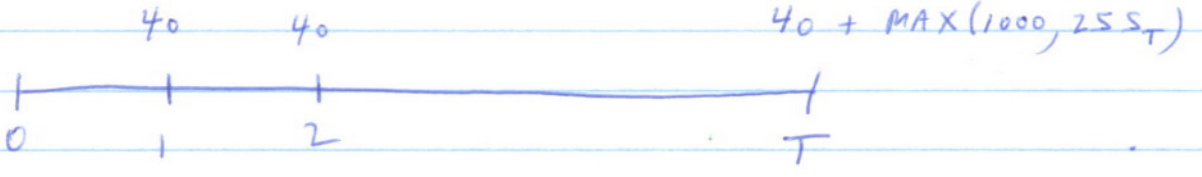
(2) $30 \cdot 37 = 1110$

[General CF formula:

$$CF(S_T) = 30 S_T - 30(S_T - 40)_+ + 20(S_T - 60)_+$$

$$\therefore CF(37) = 30 \cdot 37 = 1110 \,]$$

(3) The cash flows from the convertible bond are:

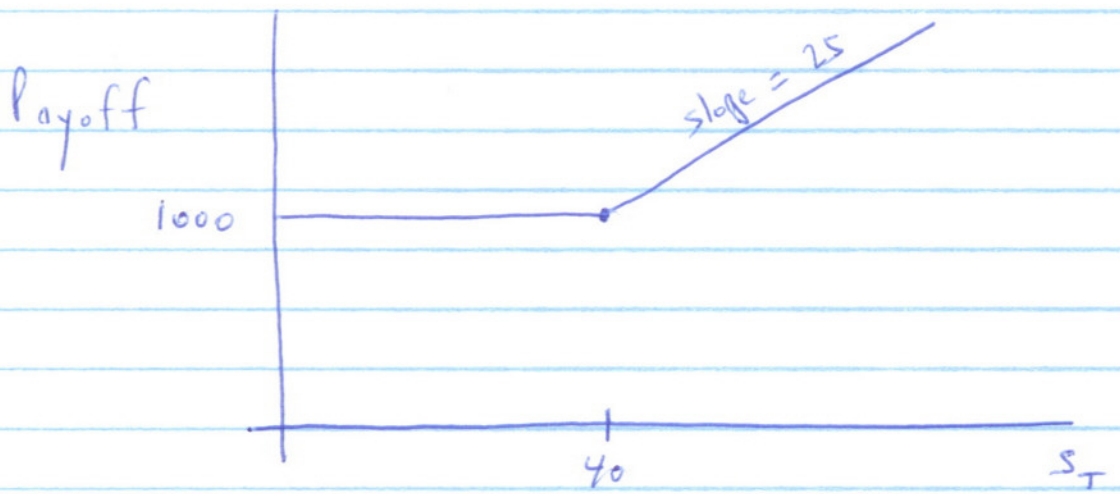


The bondholder will choose 1000 or 25 shares at bond maturity, whichever is larger, and hence the final payment of $MAX(1000, 25S_T)$.

$$MAX(1000, 25S_T) = 1000 + (25S_T - 1000)_+$$

$$= 1000 + 25(S_T - 40)_+$$

$\therefore$ the convertible bond is a 4% annual coupon bond with a \$1000 face amount PLUS 25 call options on RBC stock expiring at the same time as the bond matures with an implicit strike price of \$40.



Q2:

"Quick": Both options have 0 value at expiration. ∴ your gain or loss depends on accumulated value of option premiums.

$$-18(1.005)^6 + 44(1.005)^6 = 26.79$$

26.79 Gain

Alternative: Profit is:

$$(830 - S_T)_+ - (S_T - 830)_+ - 18(1.005)^6 + 44(1.005)^6$$

$$= 830 - S_T - 18(1.005)^6 + 44(1.005)^6$$

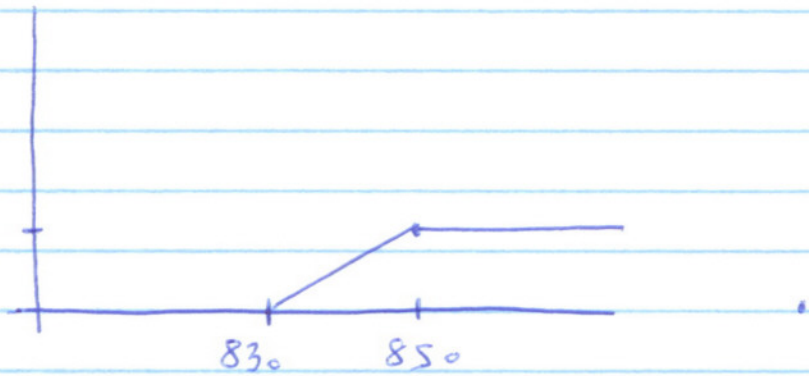
∴ for $S_T = 830$ you get:

26.79 Gain

[Note: $(830 - S_T)_+ - (S_T - 830)_+ = 830 - S_T$

is a synthetic short position in the forward contract.]

Q3: The cash flow from being long the 830-call and short the 850-call is:



The total profit at the end of 6 months is the cash flow less the accumulated value of net option premiums.

$$\text{Profit} = (S_T - 830)_+ - (S_T - 850)_+ - [C(830) - C(850)] \cdot (1.005)^6.$$

The most the investor can lose is the accumulated value of the net option premiums which occurs when $S_T \leq 830$.

$\therefore$ Answer = Max Loss of $[C(830) - C(850)](1.005)^6$

but since $C(830)$ & $C(850)$ are not given the number cannot be computed.

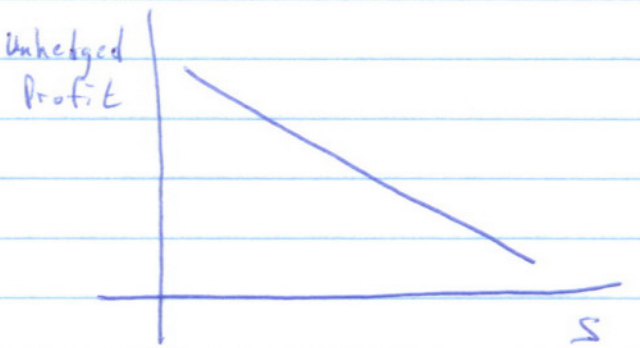
$\therefore$ any of the answers could be true, but your reasons must be given.

Q4:

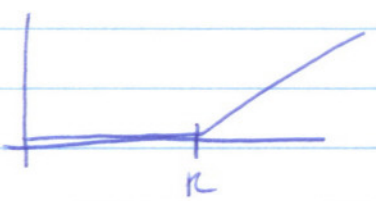
"Short Answer": The long call strategy allows the manufacturer to benefit from price declines while still maintaining a hedge (i.e. protection) against price increases. [A SHORT ANSWER IS FINE.]

"Full Analysis":

Since the hedger is a manufacturer, we must have an unhedged profit chart such as:



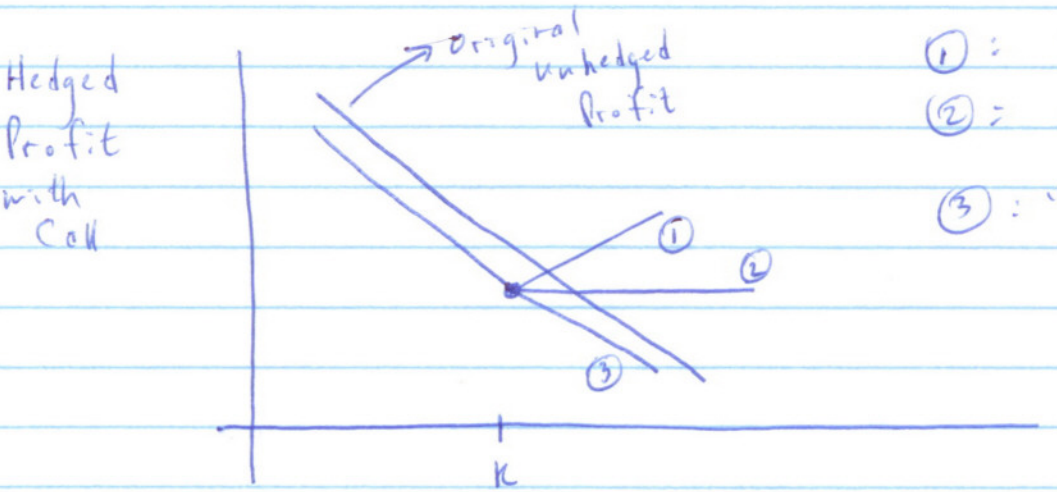
A call has a cash flow patter like:



and a forward contract has a cash flow pattern:

The shape of the hedged profit with a call or a forward depends on the number of contracts used.

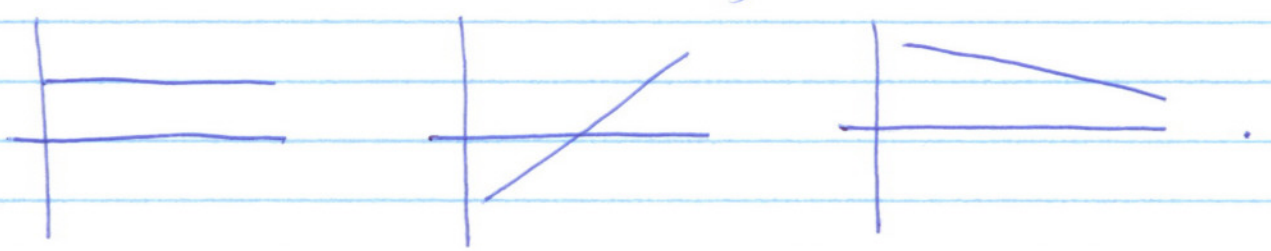
For a hedged profit with a call, we could have a picture like:



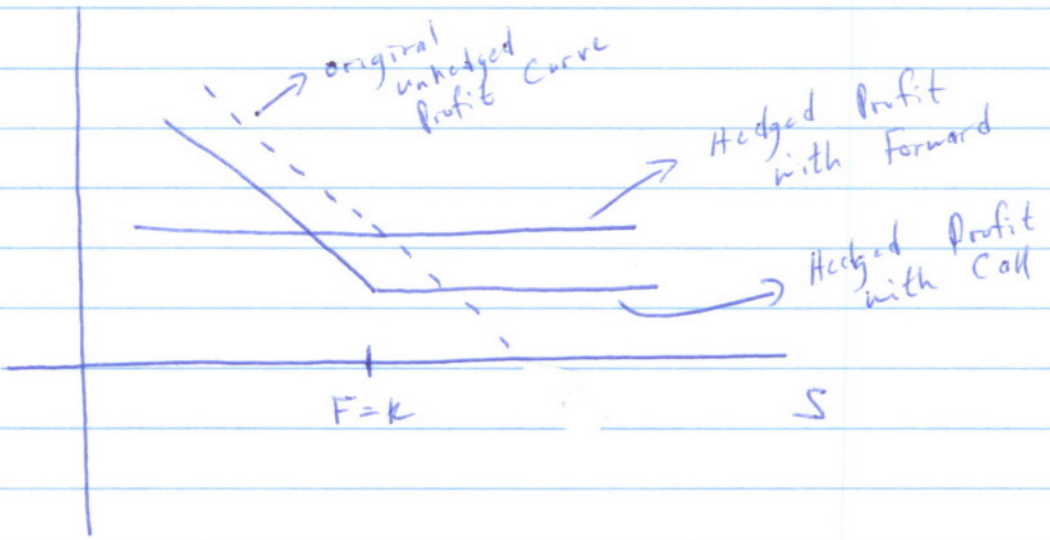
① : "TOO MANY" CALLS
② : EXACT CONSTANT HEDGE
③ : "TOO FEW" CALLS

The shift down from the original unhedged profit depends on the actual call premium.

For a hedged profit with a forward contract we can have a flat hedged profit curve or one with +ve or -ve slope depending on how many forward contracts are bought:



IF CALL STRIKE = FORWARD PRICE (i.e. $K = F$) AND $S_T > F$ THEN THE FORWARD HEDGE WILL TURN OUT BETTER THAN THE CALL HEDGE BECAUSE OF THE COST OF THE CALL PREMIUM.



Q5:

Profit Before Tax

$= 4{,}000{,}000(2.40) - 9{,}200{,}000 = 400{,}000$

Tax Due $= 400{,}000(.25) = 100{,}000$

Net Gain $= 400{,}000 - 100{,}000 = 300{,}000$.

Answer = 300,000.

Alternative:

Pre-Tax Unhedged Profit $= 4{,}000{,}000(2.1) - 9{,}200{,}000$
$= -800{,}000$

Profit on Hedge (Pre-Tax) $= 4{,}000{,}000(2.4 - 2.1) = 1{,}200{,}000$

Net Pre-Tax Profit $= 400{,}000$

Net Gain $= 400000(1 - .25) = 300{,}000$.

Q6:

The price of the butterfly-spread is $713.07 - 2 \times 496.46 + 333.96 = 54.11$.

[See pg 83 of text.]

Q7:

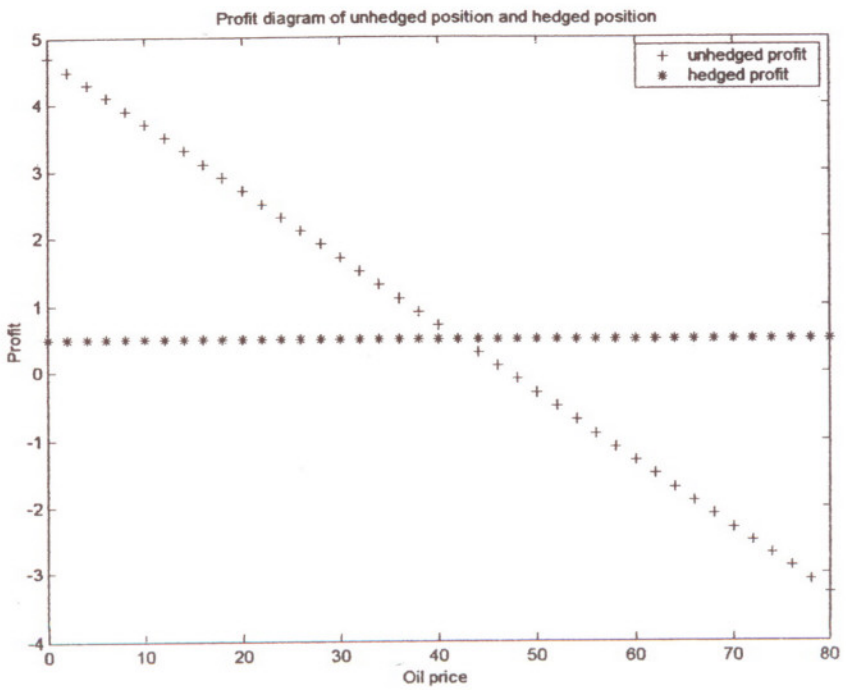
We have seen in question 4.3 that the profit of Plastics Corp. can be described as:

$$P_{plastics} = \$4.70 - 0.1 \times p_{oil}$$

Therefore, we will need to buy 1/10 of a futures contract to completely remove the oil price risk from Plastics Corp.'s profits.

Oil Price in One Year	Unhedged Profit	Profit on 1/10 Long Forward	Hedged Profit
\$25	\$2.20	–\$1.70	\$0.50
\$30	\$1.70	–\$1.20	\$0.50
\$35	\$1.20	–\$0.70	\$0.50
\$40	\$0.70	–\$0.20	\$0.50
\$45	\$0.20	\$0.30	\$0.50
\$50	–\$0.30	\$0.80	\$0.50

We obtain the following profit diagrams:



Q8:

In this exercise, we need to first find the future value of the call premia. For the \$35-strike call, it is: $\$8.085 \times 1.0408 = \$8.415$. The following table shows the profit calculations of the \$35.00-strike call and for one unit of resin. The calculations for the other call is similar.

Oil Price in One year	Unhedged Profit	Profit on 1/10 Long \$35.00-Strike Call	Call Premium	Net Income on Hedged Profit
\$25	\$2.20	0	\$0.8415	\$1.3585
\$30	\$1.70	0	\$0.8415	\$0.8585
\$35	\$1.20	0	\$0.8415	\$0.3585
\$40	\$0.70	\$0.50	\$0.8415	\$0.3585
\$45	\$0.20	\$1.00	\$0.8415	\$0.3585
\$50	–\$0.30	\$1.50	\$0.8415	\$0.3585

We obtain the following profit diagrams: